When the Prandtl number of the solution falls below 150 (for Re = const and the geometric conditions determined by the symmetry of the process) the considerable reduction in the rate of the reaction may be compensated only by increasing the initial velocity of the jets of solution very considerably in comparison with that for solutions with $Pr \geq 150$, which leads to engineering difficulties and is fairly uneconomic.

Thus, the general form of the solution of the mass-transfer equation for a metallic surface dissolving in turbulent jets of aqueous solutions and the analysis based on it are confirmed by experiment. This provides a basis for the choice of specific solutions such that the metallic surface dissolves most efficiently under the action of turbulent jets, and also the possibility of evaluating the suitability of new solutions.

## NOTATION

$\nu_i$, stoichiometric coefficients; $\beta$, diffusion-rate constant; D, diffusion coefficients; $\nu$, kinematic viscosity of solution; $d_{eff}$, effective diameter of atomizer output nozzle; R, size (radius) of reaction surface; $V_x = v_x/U_0$, $V_y = v_y/U_0$, dimensionless velocity components of solution at reaction surface; $U_0$, initial velocity of jets on emission from atomizer; $C = c/C_0$, concentration at reaction surface, the ratio of the local value to the value $C_0$ far from the reaction surface; $X = x/(b/2)$, $Y = y/(b/2)$, dimensionless coordinates at reaction surface; b/2, half-width of jets of solution at reaction surface; $Nu = (\beta \bullet R)/D$, diffusional Nusselt number; $Pe = (U_0 \bullet R)/D$, diffusional Peclet number; $Pr = \nu/D$, Prandtl number; $Re = (U_0 \bullet d_{eff})/\nu$, $Re_v = (v \bullet R)/\nu$, Reynolds numbers referred to the emission velocity of the jets and their velocity at the reaction surface; $St_v = \beta/v$, Stanton number; $G_0$, flow rate of solution at atomizer outlet; A, half-amplitude of motion of atomizer with respect to the reaction surface; $t/t_d$, ratio of the residence time of the surface in the jets to the time at which the surface is completely dissolved.

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# CALCULATION OF FLOW OF A MEDIUM IN THE GAP BETWEEN A ROTATING DISK AND A FIXED BOUNDARY WALL


M. I. Tsaplin



UDC 532.526



This paper examines the problem of determining the friction coefficients on the side surface of a rotating disk and a fixed boundary wall for different directions of the flowing medium.


Article [1] gave theoretical relations which can be used to calculate the swirl of a medium flowing in the gap between a rotating disk and a fixed boundary wall. In performing such calculations one must know the friction coefficients on the disk and the wall. It is usually assumed that these coefficients can be determined from known empirical relations for a rotating disk and an infinite space [2], if the Reynolds numbers for the disk and the wall are calculated using the relative velocity, i.e.,

$$\left.\begin{aligned} \xi &= 0.0535\,\mathrm{Re}^{-0.2} \quad \text{for} \quad \mathrm{Re} \geqslant 3.1\cdot 10^5, \\ \xi &= 1.234\,\mathrm{Re}^{-0.5} \quad \text{for} \quad \mathrm{Re} < 3.1\cdot 10^5, \end{aligned}\right\} \tag{1}$$


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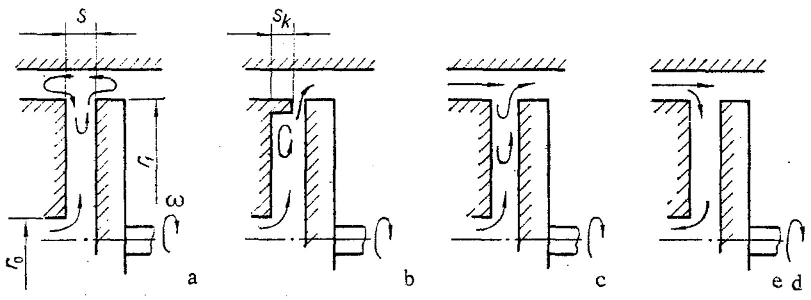


Fig. 1. Different flow schemes in the cavity around the rotating disk.

TABLE 1. Correction Factors at Different Radii

| $x$ | $Re_d \cdot 10^{-5}$ | $k_d$ | $Re_w \cdot 10^{-5}$ | $k_w$ | $\Delta\bar{P}_x$, % | $\Delta\bar{V}_u$, % |
|---|---|---|---|---|---|---|
| 0,650 | 7,156 | 1,013 | 2,904 | 1,963 | 0,024 | 0,080 |
| 0,700 | 8,221 | 0,972 | 3,457 | 2,172 | 0,018 | 0,077 |
| 0,750 | 9,362 | 0,992 | 4,058 | 2,490 | 0,022 | 0,079 |
| 0,800 | 10,618 | 0,996 | 4,672 | 2,800 | 0,025 | 0,186 |
| 0,850 | 11,983 | 1,040 | 5,301 | 3,260 | 0,026 | 0,292 |

where $Re = Re_d = (u - v_u)r/\nu$ for the disk and $Re = Re_w = v_u r/\nu$ for the wall.

Comparison of the results using Eq. (1) and experimental data show that there is appreciable divergence in the velocity distribution and the static pressure. This divergence exists over the whole range of variation of rotational speed of the disk, flow rate of the medium, various flow directions, and values of relative gap investigated.

To analyze the causes of this discrepancy it was postulated that the velocity and pressure distributions along a radius can be affected by secondary flows arising on the fixed wall. It is known that secondary flows exist for centrifugal (from center to periphery) and centripetal (from periphery to center) flow directions. These secondary flows cause a pumping action of the disk and are very clearly evident experimentally [3, 4]. In this regard there is practical interest in calculating the friction coefficients on the wall and the disk for a measured distribution of static pressure along the radius and measured speed of rotation of the medium in the middle of the gap.

To solve this problem one must write down the equations of motion in the circumferential and radial directions. Without dwelling in detail on the derivation of the original equations, which differ very little from those derived in [1], we shall write a system of equations for flow of a compressible fluid in a gap of constant width. These equations are more convenient in dimensionless form, as follows:

$$\frac{d}{dx}(x^2\bar{V}_u) = \frac{2\pi\omega r_1^3\rho}{m_s}\left[\pm k_d\xi_d\frac{(x^2 - x^2\bar{V}_u)^2}{2} \mp k_w\xi_w\frac{x^4\bar{V}_u^2}{2}\right], \tag{2}$$

$$\frac{d\bar{P}_x}{dx} = \frac{K\mathrm{M}_r^2\bar{P}_x}{(1-K\mathrm{M}_r^2)}\left[\frac{\mathrm{M}_{u,1}^2}{\mathrm{M}_r^2}\,\frac{(x^2\bar{V}_u)^2}{x^3} \pm \frac{1}{x}\right]. \tag{3}$$

Here the following notation is used:

$$\bar{V}_u = v_u/\omega r,\ \bar{P}_x = p/p_1,\ \mathrm{M}_r = v_r/a_S,\ \mathrm{M}_{u,1} = \omega r_1/a_S.$$

In Eqs. (2) and (3), the superscripts on the right correspond to centrifugal flow with initial value $\bar{V}_{u,0} < 1$, while the subscripts correspond to centripetal flow with $\bar{V}_{u,1} < 1$. It should be noted that in the experiment performed $\bar{V}_{u,0}$ and $\bar{V}_{u,1}$ were always less than 1, and therefore cases $\bar{V}_{u,0} > 1$ and $\bar{V}_{u,1} > 1$ are not considered here.

These equations were used to calculate correction factors for the friction coefficients $\xi_d$ for the disk and $\xi_w$ for the fixed wall, computed from Eq. (1).

An experimental determination of the velocity and the static pressure was made on a facility described in [3]. It consisted of a plane disk located near a wall with a certain gap s constant along a radius (Fig. 1). The disk diameter corresponded to the peripheral

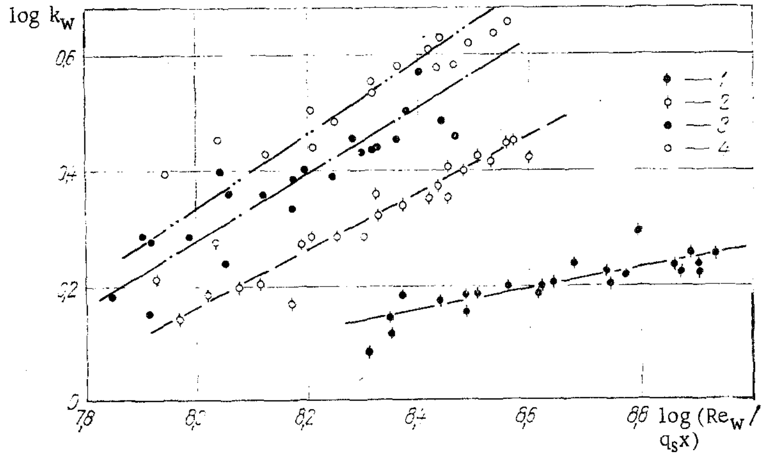

Fig. 2. The correction factor to the drag coefficient on the fixed wall for centrifugal flow with an open axial gap: 1) $s/r_1 = 0.03$; 2) 0.04; 3) 0.05; 4) 0.075.

TABLE 2. Correction Factors at Various Radii for Centripetal Flow ($s/r_1 = 0.05$, $n = 10{,}000$ rpm, $q_{s,1} \cong 0.8 \cdot 10^{-3}$)

| $x$ | $Re_d \cdot 10^{-5}$ | $k_d$ | $Re_w \cdot 10^{-5}$ | $k_w$ | $\Delta\bar{P}_x$, % | $\Delta\bar{V}_u$, % |
|---|---|---|---|---|---|---|
| 0,600 | 7,048 | 1,001 | 8,076 | 1,758 | 0,000 | 0,394 |
| 0,650 | 8,486 | 1,003 | 9,346 | 1,710 | 0,027 | 0,530 |
| 0,700 | 10,053 | 1,001 | 10,733 | 1,762 | 0,001 | 0,596 |
| 0,750 | 11,432 | 1,001 | 12,549 | 1,675 | 0,009 | 0,849 |
| 0,800 | 12,848 | 1,001 | 14,592 | 1,475 | 0,050 | 0,677 |

size of the wall and was 400 mm. On the facility one could vary the gap from 4 to 50 mm and the rotational frequency from 2000 to 10,000 rpm; also one could vary the flow direction. The air speed was measured using a calibrated orifice plate, located in the middle of the gap, and by means of a special mechanism for traversing along a radius. The static pressure was measured on the wall at 12 points along a radius by means of a water U-tube manometer. In addition, measurements were made of the air flow rate, the air temperature in the gap, and the rotational speed of the disk.

To determine the local correction factors $k_d$ and $k_w$, the whole interval in which the velocity and static pressure measurements were taken was divided into 11 sections. Within each section the quantities $k_d$ and $k_w$ were determined from the condition that the corresponding theoretical values for the measured initial values $\bar{V}_{u,0}$ and $\bar{P}_{x,0}$ at the end of each section should coincide with the measured values to within a preset accuracy. The accuracy with which the calculations of $k_d$ and $k_w$ were made led to agreement between the measured and calculated values of velocity and pressure at the end of the section to within 1% of the corresponding relative values.

The problem was solved on a computer using a Runge—Kutta method for numerical integration of Eqs. (2) and (3). Here one should note that essentially the problem stated in this way reduces to solution of a system of two nonlinear equations. This solution can be implemented most efficiently by looking for a minimum of the following function:

$$y = \Delta\bar{P}_x^2 + \varepsilon\Delta\bar{V}_u^2, \tag{4}$$

where $\Delta\bar{P}_x$ and $\Delta\bar{V}_u$ denote the allowable values of discrepancies between the measured and calculated values at the end of the section investigated, for the relative pressure and flow speed, respectively, and $\varepsilon$ is a constant factor, which serves to equalize the weight of $\Delta\bar{P}_x$ and $\Delta\bar{V}_u$ in order to achieve the desired accuracy.

Table 1 shows the results of analyzing experiments using the above method for centrifugal flow with $s/r = 0.05$, $n = 10{,}000$ rpm, and a dimensionless mass flow at radius $r_1$ of $q_{s,1} = 1.38 \cdot 10^{-3}$. It can be seen that the correction factor $k_d$ is practically 1. This means that the local friction coefficient $\xi_d$ on the disk can be calculated from Eqs. (1). A similar result was found over the whole range investigated of variation in the gaps between the wall and the disk, the speed of rotation, and the air flow rate.

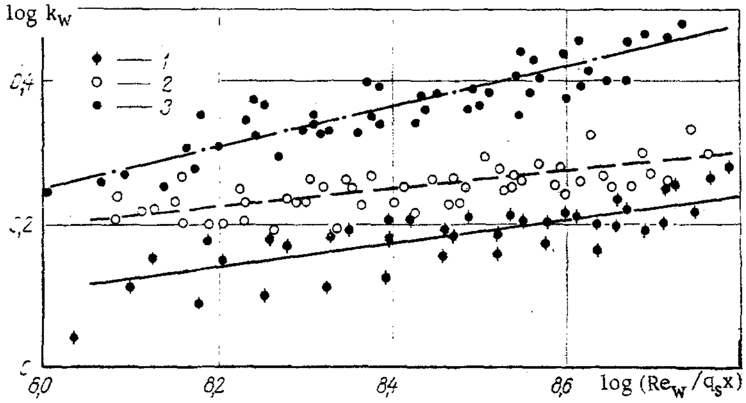


Fig. 3. The correction factor to the drag coefficient on the fixed disk for centrifugal flow with a deflector plate at the peripheral radius ($s_p/r_1$ = 0.045): 1) $s/r_1$ = 0.05; 2) 0.06; 3) 0.075.

Quite a different picture was obtained from analysis of the quantity $k_w$. In the case of centrifugal flow with an open axial gap at the peripheral radius between the boundary wall and the rotating disk (Fig. 1a), the correction factor can vary over quite wide limits, there being smaller values of $k_w$ for relatively small air flow rates close to the axis of rotation and larger values at the peripheral radii.

This marked feature in the variation of $k_w$ can be explained by the fact that secondary flows arising at the peripheral radius along the fixed wall do not have initial swirl, since the air arrives through the open gap from the annular space above the disk, in which there is practically complete stagnation of the circumferential velocity component because of the large dimensions of the surfaces. In the one-dimensional model considered the interaction of the secondary flow with air moving in a radial direction, and deflected by the rotating disk, is equivalent to an apparent increase in the friction coefficient on the fixed wall.

This experimental investigation and the subsequent data reduction yield values of $k_c$ corresponding to various values of s, n, and $m_s$. However, in this form the data on $k_w$ are unsuitable for practical use. In this regard there is a definite practical interest in obtaining a generalized law which one could use to relate $k_w$ and the above parameters.

From analysis of the kinematic similarity conditions in the cavity near the disk we can determine the following functional relation:

$$k_w = f(\mathrm{Re},\ v_u/v_r,\ s/r_1).$$

For $s/r_1$ = idem the experimental data can be correlated satisfactorily by a single relation, if we take the group $Re_w/q_s x$ as the governing parameter, where $q = m_s/\rho\omega r^3$ is the dimensionless flow rate of the medium at the current radius. The results of this data analysis are shown in Fig. 2 in logarithmic coordinates for several values of the relative gap. It can be seen that the slope of the experimental relations increases as $s/r_1$ increases, which is evidence of an increase in the drag coefficient on the fixed wall due to a downflow of large masses of nonswirling air into the inside cavity. The data shown in Fig. 2 can be approximated by the following relation:

$$k_w = c\left(\frac{\mathrm{Re_w}}{q_s x}\right)^m, \tag{5}$$

where

$$\log c = 1.54 + 7.76(s/r_1 - 0.03)^{0.275},$$
$$m = 0.2 + 1.205(s/r_1 - 0.03)^{0.3}.$$

This relation is valid for $0.03 \leqslant s/r_1 \leqslant 0.075$; $0.5\cdot 10^{-3} \leqslant q_s \leqslant 10^{-2}$; $5\cdot 10^5 \leqslant Re_w \leqslant 2.5\cdot 10^6$.

It should be noted that in this data reduction the friction coefficients on the wall and the disk were computed using a relation appropriate for Re > $3.1\cdot 10^5$. An attempt to reduce the experimental data, allowing for transition of the turbulent regime to a laminar one at

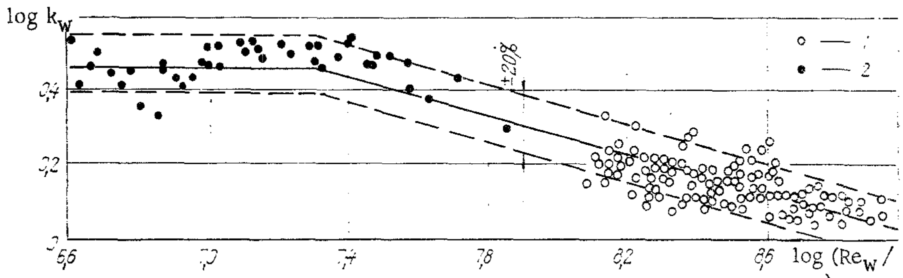


Fig. 4. The correction factor to the drag coefficient on the fixed wall for centripetal flow: 1) the present data; 2) data of [4].

local values of Re less than $3.1 \cdot 10^5$, led to a large scatter in the points. At a critical Reynolds number, apparently, the fixed boundary wall has an effect. A decrease in the critical Reynolds number with a wall present was also noted in [5-7].

With a closed axial gap (Fig. 1b), or with swirling gas flow on the periphery of the disk (Fig. 1c), the flow picture may be different in the cavity near the disk. In the cooling arrangements for turbine disks the axial gap, as a rule, is closed by shelf plates [8] or is sealed by a special labyrinth. Therefore, it is of great practical interest to investigate flow in the cavity near the disk, when there is a special deflector plate at the peripheral radius on the fixed wall, preventing free flow of air from the space above the disk (Fig. 1b).

Experiments show that, with a constant air flow rate directed from the center to the periphery, the presence of a deflector plate promotes large swirl of the flow in the cavity, compared with the case considered above. Reduction of the experimental data has shown that the values of $k_w$ are then lowered appreciably and the effect of the group $Re_w/q_s x$ on variation of $k_w$ becomes quite negligible.

Figure 3 shows the results of tests with several values of $s/r_1$ (the relative length of the deflector plate in the axial direction was kept constant and equal to $s_p/r_1 = 0.045$). With an increase in the relative gap $s/r_1$, for constant values of $Re_w/q_s x$, a tendency for $k_w$ to increase was observed. For the range of variation of Re and $q_s$ investigated, the experimental data can be approximated, with an accuracy of ±10%, by the relations

$$k_w = 0.067(Re_w/q_s x)^{0.16} \quad \text{for} \quad s/r_1 = 0.05, \tag{6}$$

$$k_w = 0.158(Re_w/q_s x)^{0.125} \quad \text{for} \quad s/r_1 = 0.06, \tag{7}$$

$$k_w = 0.0095(Re_w/q_s x)^{0.282} \quad \text{for} \quad s/r_1 = 0.075, \tag{8}$$

which are valid in the range $0.5 \cdot 10^{-3} \leqslant q_s \leqslant 8 \cdot 10^{-3}$ and $0.5 \cdot 10^6 \leqslant Re \leqslant 2.5 \cdot 10^6$.

Similar investigations were conducted for centripetal flow along the rotating disk (Fig. 1d). The technique for determining the velocity and the static pressure was unchanged for these experiments. Reduction of the experimental data showed that the correction factor $k_d$ for centripetal flow was practically equal to 1 (Table 2), just as for the centrifugal case. For this type of flow the limits of variation of $k_w$ were considerably contracted.

A subsequent data reduction was performed to find a generalized correlation which would allow the basic kinematic parameters describing the flow in the cavity near the disk to be related with $k_w$. For a variation in relative gap from 0.03 to 0.075 the data-reduction correlation used for the centrifugal flow was found to approximate the experimental data by a single relation (Fig. 4) to within ±20%:

$$k_w = \frac{263}{(Re_w/q_s x)^{0.266}}. \tag{9}$$

To achieve greater generality in the results obtained we attempted to reduce the experimental data presented in [4]. That paper contained information on the distribution of relative velocity $\bar{V}_u$ of the flow core and the static pressure coefficient $\psi = p/\rho u_1^2$ along the radius of a disk rotating near a fixed wall, with various flow directions of an incompressible fluid. Because the quantity $k_w$ is appreciably affected in centrifugal flow by the presence of a deflector plate or a housing, and the geometry was not clarified in [4], the data processing was performed only for the centripetal flow.

For an incompressible fluid in a gap of constant width the original equations of motion in the circumferential and radial directions take the form

$$\frac{d}{dx}(x^2\bar{V}_u) = \frac{1}{q_{s,1}}\left[k_d\xi_d\frac{(x^2 - x^2\bar{V}_u)^2}{2} - k_w\xi_w\frac{x^4\bar{V}_u^2}{2}\right], \tag{10}$$

$$\frac{d\psi}{dx} = \frac{1}{x^3}\left[(x^2\bar{V}_u)^2 + q_{s,1}\left(\frac{r_1}{2s}\right)^2\right]. \tag{11}$$

Here $q_{s,1} = m_s/\rho\omega r_1^3$ is the dimensionless flow rate at radius $r_1$. This parameter was varied from $10^{-3}$ to $2.73 \cdot 10^{-3}$. The friction coefficients $\xi_d$ and $\xi_w$ were calculated according to the Reynolds number Re, which was $0.87 \cdot 10^6$ in the experiment, and the relative swirl $\bar{V}_u$. The experimental investigation in [4] was carried out at two values of $s/r_1$, 0.056 and 0.15, respectively.

The results of processing these experiments are shown by the solid points in Fig. 4. It can be seen that these data confirm the general trend in the variation of $k_w$ with reduction of $Re_w/q_s x$, where, beginning at a certain value of $Re_w/q_s x$, $k_w$ remains practically constant. Thus, from these data one can postulate that the empirical relation (9) will be valid in the following range of variation of the geometric and kinematic parameters: $0.03 \leqslant s/r_1 \leqslant 0.15$ and $2 \cdot 10^7 \leqslant (Re_w/q_s x) \leqslant 1.25 \cdot 10^9$. For $(Re_w/q_s x) < 2 \cdot 10^7$, $k_w \cong 3$, and for $(Re_w/q_s x) \geqslant 1.25 \cdot 10^9$, $k_w \approx 1$.

The data obtained can be used to calculate flow in the cavities around a rotating disk in the presence of flow of a coolant or gaseous medium. In the general case, when one must account for variation in the friction coefficients on the wall and the disk, as well as the nonisothermal nature of the flow, arising from heat transfer and frictional work, the original equations describing the motion of a compressible medium in the circumferential and radial directions can be solved numerically.

## NOTATION

$x$, dimensionless radius; $r_1$, greatest radius, along which flow occurs; $r$, current radius; $s$, gap between the disk and the wall; $s_p$, length of the deflector plate closing the axial gap at radius $r_1$; $q_{s,1}$, dimensionless flow rate at radius $r_1$; $q_s$, dimensionless flow rate at the current radius; $m_s$, mass flow rate of the medium; $\bar{V}_u$, relative velocity of swirl of the medium at radius $x$; $\bar{V}_{u,1}$, initial relative velocity with centripetal flow; $\bar{V}_{u,0}$, initial relative velocity in centrifugal flow; $\bar{P}_x$, relative pressure at the current radius; $\bar{P}_{x,0}$, initial relative pressure; $p$, static pressure at the current radius; $p_1$, static pressure at radius $r_1$; $a_s$, speed of sound; $\rho$, density of the flowing medium; $\xi_w$, friction coefficient on the wall; $\xi_d$, friction coefficient on the disk; $k_w$, $k_d$, correction factor to the friction coefficient for the wall and disk, respectively; $\omega$, angular velocity; $n$, frequency of rotation; $k$, adiabatic index; $m$, $c$, coefficients in the approximations; $y$, auxiliary function used to calculate the correction factors; $Re_d$, $Re_w$, local relative Reynolds numbers for the disk and wall, respectively; Re, Reynolds number based on the circumferential speed of the disk at radius $r_1$; $M_{u,1}$, Mach number based on the circumferential speed of the disk at the peripheral radius; $M_r$, Mach number based on the radial velocity of the medium in the gap; $\varepsilon$, equalizing coefficient; $\Delta\bar{P}_x$, $\Delta\bar{V}_u$, relative disagreement between the measured and calculated values of the pressure and the medium swirl velocity.

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